Load Effect Envelopes for Continuous Structures by Influential Superposition

by
T. Bartlett Quimby[1]

Abstract

Finding the envelope of maximum shear and moment in a structure is required to ensure that adequate strength is provided in the members. For continuous structures subjected to loads that may be placed everywhere, anywhere, or nowhere the process of developing these envelopes may be quite involved. The process for finding these maximum values typically involves the development of a series of influence lines that indicate where to place live loads so as to cause maximum effects. For continuous structures, many influence lines may need to be developed in order to guarantee that all appropriate load arrangements are accounted for. This can lead to quite a few load arrangements, each requiring an analysis of the structural system.

Determining the envelope values by the method described in this paper can reduce the number of load cases that need to be considered when considering live load distribution. The method is applicable to linear elastic analysis only. It makes extensive use of superposition principles and inherently considers only those load cases that contribute to maximum and minimum load effects at any given location on a structure.

Introduction

One of the most tedious tasks of structural analysis is to determine the envelope of values for shear and moment. The process requires determining the arrangement of live loads for maximum effects, analyzing the structure for the resulting load arrangements, and then comparing the results to get the envelope values. The process may lead to errors if the engineer is not diligent enough to analyze all the appropriate load arrangements. The process can also lead to an explosion of load arrangements in the interest of thoroughness.

In many cases, this process can be simplified by making a few observations when the principle of superposition may be applied and the influence of individual loads is considered. The method of influential superposition involves structural analysis under unit loads applied to discrete regions of a structure then combining the results at each location on the structure by superposition according to the influence that the loading system has on the location of interest.

The Method

As with other methods, the structural geometry is defined and the possible locations and nature (i.e. point, uniform, linearly varying, etc.) of the live loads is determined. For the method of influential superposition, unit loads of the same nature as the real live loads are applied individually to the structure and the effects on the structure are determined. The load effects may be shear, moments, axial forces, and/or deflections. Finding the upper envelope load effect value $U_{upper,j}$ on the structure at any given point, i, is found simply by adding to the dead load effect all the positive effects from the various analyzes according to equation (1).

$$U_{upper,j} = U_{DL,j} + \sum_{i=1}^{n} \max(0, P_i * U_{ij}) \qquad \textbf{(1)}$$

Where P_i is the actual load magnitude that corresponds to unit load *i* and U_{ij} is the load effect due to unit load *i* at point *j*. Similarly, to find the lower envelope value

[1] Assoc. Prof. of Civil Engrg., University of Alaska Anchorage, School of Engrg, 3211 Providence Drive, Anchorage, AK 99508. Phone: (907) 786-1046. E-mail: aftbq@uaa.alaska.edu.

$U_{lower,j}$ is found by adding to the dead load effect all the negative effects from the various analyzes according to equation (2).

$$U_{lower,j} = U_{DL,j} + \sum_{i=1}^{n} \min(0, P_i * U_{ij}) \qquad \textbf{(2)}$$

Example Problem

The method is best understood by application to a example problem. In the interest of space, a simple continuous beam problem with uniformly distributed loads is chosen and the envelope values will be determined only at four points along each span, in addition the values at the supports. Geometry of the problem is shown in Figure 1. The sixteen points along the beam represent location where envelope values will be determined.

A uniformly distributed dead load of 4 kN/m will be used for all spans. Uniformly live loads of 6 kN/m, 8 kN/m, and 7 kN/m will be used for spans 1, 2, and 3 respectively.

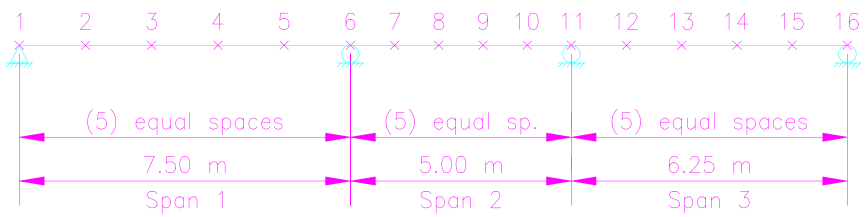

Figure 1
Example Problem Geometry

Figure 2 shows the load cases that might be considered in a typical analysis of the given structure. The result is that five analyzes must be done and the results of the live load cases must be superimposed on the dead load case so as to determine the load effects at each point of interest. For this example the load effect, U, to be considered is bending moment, M.

Figure 3 shows the load cases used by the method of influential superposition. With this method, three analyzes are run. The load cases are numbered by span. For example, LC1 is the case with the unit load only on span 1. Note that the unit load is unitless.

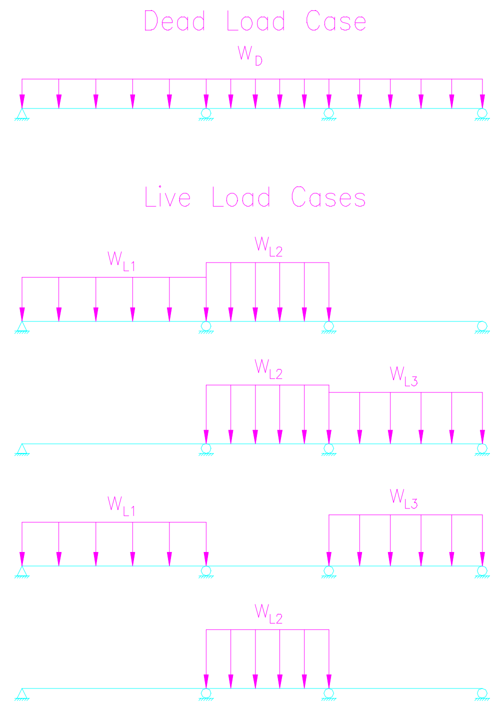

Figure 2
Typical Analysis Load Cases

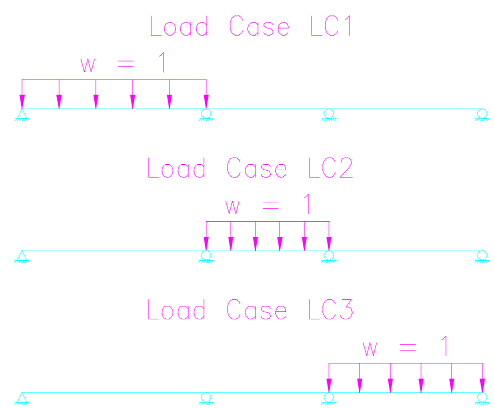

Figure 3
Influential Superposition Load Cases

The bending moment results of the three influential superposition analyzes are given in Table 1 and illustrated in Figure 4. These results can be obtained by any means of classical or numerical analysis. Since the unit load has no units, the results are in units of m^2.

Table 1
Moments resulting from unit loads

Point	LC1	LC2	LC3
	(m^2)	(m^2)	(m^2)
1	0.00	0.00	-0.00
2	3.62	-0.20	0.11
3	4.98	-0.41	0.23
4	4.10	-0.61	0.34
5	0.97	-0.81	0.45
6	-4.41	-1.02	0.57
6	-4.41	-1.02	0.57
7	-3.34	0.95	-0.11
8	-2.26	1.92	-0.79
9	-1.18	1.90	-1.48
10	-0.10	0.87	-2.16
11	0.98	-1.16	-2.84
11	0.98	-1.16	-2.84
12	0.78	-0.93	0.85
13	0.59	-0.70	2.98
14	0.39	-0.47	3.55
15	0.20	-0.23	2.56
16	-0.00	0.00	-0.00

Three combinations are now required. The first is the combination for the dead load case and the other two are the lower and upper bounds of dead plus live load.

The dead load case is computed by equation 3. Equation 3 considers the moment contribution of loads on all of the spans. Note that the equation can be simplified

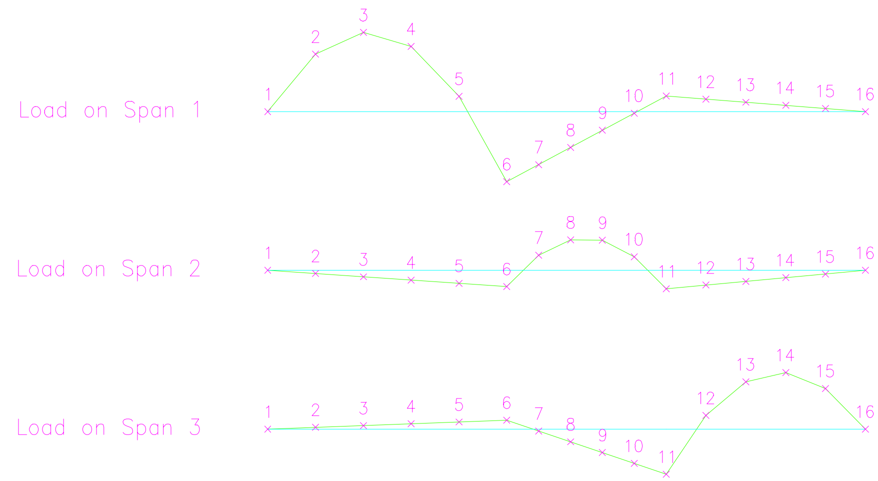

Figure 4
Moment Diagrams for the Three Load Cases

$$M_{DL,i} = w_{DL,1} * M_{1,i} + w_{DL,2} * M_{2,i} + w_{DL,3} * M_{3,i}$$
$$M_{DL,i} = \sum_{j=1}^{3} w_{DL,j} * M_{j,i} \qquad (3)$$

by bringing $w_{DL,j}$ out of the summation for the sample problem since the dead load is the same magnitude on all spans. The lower and upper bounds, $M_{lower,i}$ and $M_{upper,i}$, are determined from equations 2 and 3. Example Calculation #1 illustrates the computation of the combined value calculation at the support at point 6 (i.e. for i = 6).

Note how the live load magnitudes are applied. To get the moment at point i due to load case LCj, multiply the moment at point i due to the unit load in LC_j by the actual load applied at the same location as the unit load used in LCj. In the above sample, a unit load on span 1 (LC1) causes a moment of -4.41 m^2 at point 6. To get the moment due an applied load of 6 kN/m on span 1, multiply the moment from the unit load by the applied load. Similarly to get the moment contribution at point 6 due to an applied load of 8 kN/m on span 2, multiply the moment resulting from the unit load (-1.02 m^2) by the actual load applied to the span (8 kM/m).

$$M_{DL,6} = 4kN/m * [-4.41m^2 - 1.02m^2 + 0.57m^2]$$
$$\mathbf{M_{DL,6} = -19.44kN{\cdot}m}$$
$$M_{upper,6} = M_{DL,6} + 6kN/m * [0.00m^2] + 8kN/m * [0.00m^2] + 7kN/m * [0.57m^2]$$
$$M_{upper,6} = -19.44kN{\cdot}m + 0.00kN{\cdot}m + 0.00kN{\cdot}m + 3.99kN{\cdot}m$$
$$\mathbf{M_{upper,6} = -15.45kN{\cdot}m}$$
$$M_{lower,6} = M_{DL,6} + 6kN/m * [-4.41m^2] + 8kN/m * [-1.02m^2] + 7kN/m * [0.00m$$
$$M_{lower,6} = -19.44kN{\cdot}m - 26.46kN{\cdot}m - 8.16kN{\cdot}m - 0.00kN{\cdot}m$$
$$\mathbf{M_{lower,6} = -54.06kN{\cdot}m}$$

Example Calculation #1

M_{DL}, M_{upper}, & M_{lower} at point 6

Examine the equation for computing the dead load moment. Note that all load cases are contributing. This is true since the dead load is always everywhere present at the same time and all spans contribute to the moment at point 6.

Examining the equation for computing the upper bound, note that the terms associated with load on spans 1 and 2 are zero. This results from the maximum statement in equation 1. The moment at point 6 due to the load on spans 1 and 2 are negative and, as such, do not contribute to the upper bound moment. Similarly for the lower bound calculation, the term associated with load on span 3 is taken as zero since the resulting moment for the associated load case is positive and does not contribute to the lower bound. The end effect is that only spans 1 and 2 are loaded with live load for the lower bound and only span 3 is loaded with live load for the upper bound. These load arrangements can be predicted by drawing the influence line for moment at point 6, however the explicit step of determining the arrangements via influence line determination is not necessary since the method picks it up.

Similar calculations are made for the other points of interest. The results are tabulated in Table 2 and illustrated in Figure 5. Slight differences between that results in the sample calculation and Table 2 result from round off error. Careful observation of Figure 5 and the data in Tables 1 and 2 reveal that the upper and lower envelope values come from different live load arrangements. Other methods would require the analysis of all the different load arrangements to get the same results.

Table 2

Moment Envelope Values

Point	DL (kN-m)	Upper (kN-m)	Lower (kN-m)
1	0.00	0.00	-0.00
2	14.11	36.60	12.48
3	19.22	50.71	15.96
4	15.32	42.31	10.44
5	2.43	11.42	-4.08
6	-19.46	-15.49	-54.09
6	-19.46	-15.48	-54.09
7	-9.98	-2.36	-30.79
8	-4.51	10.89	-23.61
9	-3.03	12.13	-20.43
10	-5.56	1.37	-21.25
11	-12.08	-6.19	-41.25
11	-12.08	-6.20	-41.26
12	2.83	13.52	-4.61
13	11.50	35.92	5.92
14	13.92	41.13	10.20
15	10.08	29.16	8.22
16	-0.00	-0.00	-0.00

The same approach used here for moment envelope determination could also be

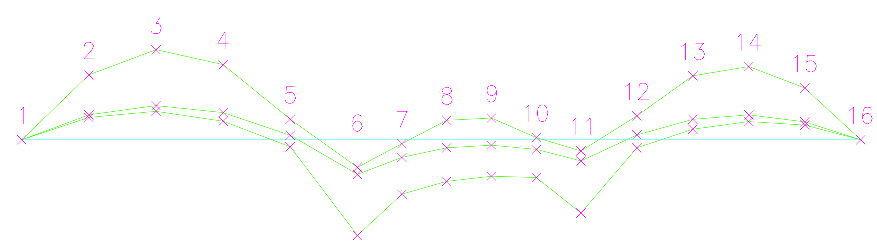

Figure 5

Example Problem Moment Envelope

used to determine shear or deflection envelopes for the same structural system. The method can also be used with arrangements of point loads and moments as well as distributed loads on more complex structures.

Conclusion

The method of Influential Superposition can be used to reduce the number of analyzes required for the determining the moment envelopes of continuous structures. The method can be easily implemented in numerical analysis programs in order to reduce the number of load cases to be developed by the user and to reduce the computational effort.

The method has the added advantage of reducing the potential for inadvertent neglect of all applicable live load arrangements.